

## Determining the Most Influential Nodes in Pinning Controllability of Connectivity Networks

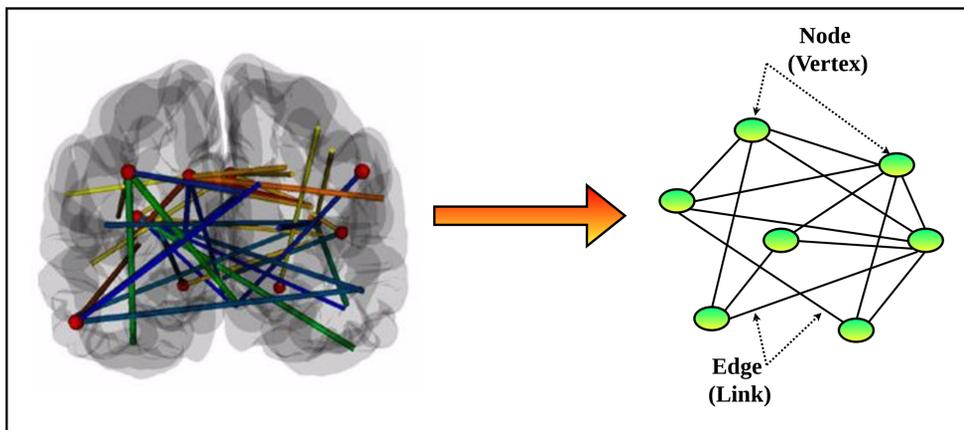


Figure: Schematic illustration of unweighted-undirected graph of complex networks in brain. Nodes or vertices can be brain regions or voxels. Edges or links are the functional or structural connections between nodes.

We consider an undirected and unweighted network  $(\mathbf{V}, \mathbf{E})$  with the set of  $\mathbf{N}$  nodes (or vertices)  $\mathbf{V}$  and a set of edges  $\mathbf{E}$ . Each node is assumed to be a dynamical system with the following dynamical equation:

$$\frac{dx_i}{dt} = F(x_i) - \sigma \sum_{j=1}^N l_{ij} H x_j \quad (1)$$

where  $x_i \in R_n$  is the  $n$ -dimensional state vector,  $F: R_n \rightarrow R_n$  defines the individual systems dynamical equation, which is considered identical for all nodes in this paper, and  $\sigma$  represents unified coupling strength.  $L = [l_{ij}] = D - A$  is the Laplacian matrix of the graph  $(\mathbf{V}, \mathbf{E})$ , where  $\mathbf{A}$  is the adjacency matrix and  $\mathbf{D}$  is a diagonal matrix of nodes degrees. Non-zero elements of  $\mathbf{H}$  determine the coupled elements of the oscillators. The pinning control objective is to synchronize all nodes to the following desired state (i.e.  $x_1(t) = x_2(t) = \dots = x_N(t) = s(t)$ ):

$$\frac{d(s(t))}{dt} = F(s(t)) \quad (2)$$

In order to pin the dynamical network [1, 2, 3] to this reference, the following control system should be designed:

$$\frac{dx_i}{dt} = F(x_i) - \sigma \sum_{j=1}^N l_{ij} H x_j + \beta_i u_i \quad i = 1, 2, 3, \dots, N \quad (3)$$

where  $u_i$  is the control signal and  $\beta_i = 1$  for driver nodes, otherwise  $\beta_i = 0$ . The system can be linearized over an equilibrium point  $x_e$  as follows:

$$\frac{dz_i}{dt} = [DF(x_e) - \sigma \lambda_i H] z_i + \beta_i u_i \quad i = 1, 2, 3, \dots, N \quad (4)$$

where  $\mathbf{D}$  stands for the Jacobian,  $\lambda_i$  is the  $i^{\text{th}}$  eigenvalue of the Laplacian matrix of the graph. In order to find the node with the most influence on pinning controllability, we restate the following definition and lemma [4].

Definition: For each node  $i$  of the undirected network  $(\mathbf{V}, \mathbf{E})$ , the Eigenratio Sensitivity Index (**ESI**) is defined as:

$$ESI(i) = [x_N^i]^2 \quad (5)$$

where  $x_N^i$  represents the  $i^{\text{th}}$  element of  $x_N$ , the eigenvector corresponds to the largest eigenvalue of the Laplacian matrix ( $\lambda_N$ ).

We apply the theoretical results on functional (<sup>18</sup>FDG-PET) and structural (MRI) connectivity graphs for CN, MCI, and AD patients. These data were obtained from the Alzheimer's Disease Neuroimaging Initiative (ADNI) database. For the structural MRI data, the connections in the graph show the inter-regional covariation of gray matter volumes in different areas while for the functional PET data, the connections do not show the correlation in activity.

## Results

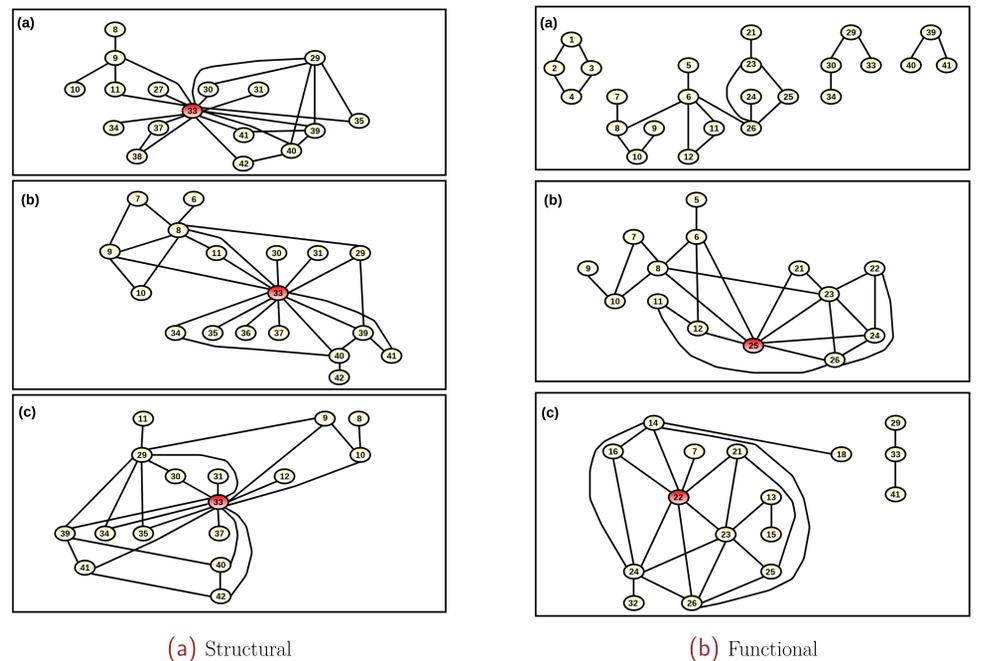


Figure: The most influential driver nodes (shown in red) in connectivity graphs for (a) CN, (b) MCI and (c) AD.

## Conclusion

Data indicate that the connections in the structural graphs illustrate the inter-regional covariation of gray matter volumes in different areas. The connections in the functional graphs do not illustrate the correlation in activity. However, they show the correlation in the glucose uptake between different regions.

## References

- [1] A. Meyer-Bäse, R. Roberts, I. Illan, U. Meyer-Bäse, M. Lobbes, A. Stadlbauer, and K. Pinker-Domenig. Dynamical graph theory networks methods for the analysis of sparse functional connectivity networks and for determining pinning observability in brain networks. *Frontiers in Computational Neuroscience*, <https://doi.org/10.3389/fncom.2017.00087>, 1 2017.
- [2] Amirhessam Tahmassebi, Katja Pinker-Domenig, Georg Wengert, Marc Lobbes, Andreas Stadlbauer, Francisco J Romero, Diego P Morales, Encarnacion Castillo, Antonio Garcia, Guillermo Botella, et al. Dynamical graph theory networks techniques for the analysis of sparse connectivity networks in dementia. In *Smart Biomedical and Physiological Sensor Technology XIV*, volume 10216, page 1021609. International Society for Optics and Photonics, 2017.
- [3] Amirhessam Tahmassebi, Katja Pinker-Domenig, Georg Wengert, Marc Lobbes, Andreas Stadlbauer, Norelle C Wildburger, Francisco J Romero, Diego P Morales, Encarnacion Castillo, Antonio Garcia, et al. The driving regulators of the connectivity protein network of brain malignancies. In *Smart Biomedical and Physiological Sensor Technology XIV*, volume 10216, page 1021605. International Society for Optics and Photonics, 2017.
- [4] A. Moradi Amani, M. Jalili, X. Yu, and L. Stone. Finding the most influential nodes in pinning controllability of complex networks. *IEEE Transactions on Circuits and Systems II*, 64:685–689, 8 2017.

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